EXPLORATION

# 12-2 Geometric Sequences

In a ***geometric sequence,*** the ratio of one term to the next is always the same.

The table shows the first four terms of a geometric sequence.

| Term 1 | Term 2 | Term 3 | Term 4 | Term 5 | Term 6 | Term 7 |
|---|---|---|---|---|---|---|
| 10 | 20 | 40 | 80 | | | |

**1.** Find the ratio between two consecutive terms.

**2.** Find the next three terms.

**3.** Find a short way to find the 20th term.

The table shows a geometric sequence with the first three terms missing.

| Term 1 | Term 2 | Term 3 | Term 4 | Term 5 | Term 6 | Term 7 |
|---|---|---|---|---|---|---|
| | | | 150 | 75 | 37.5 | 18.75 |

**4.** Find the ratio between two consecutive terms.

**5.** Find the first three terms.

## Think and Discuss

**6.** **Discuss** the similarities and differences between the two sequences.

**7.** **Explain** your strategy for finding the 20th term in the first sequence.

**This page is available as a transparency.**

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# Exploration Recording Sheet

## *Geometric Sequences*

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| 10 | 20 | 40 | 80 | | | |

1. Find the ratio between two consecutive terms. _____
2. Find the next three terms.
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The table shows a geometric sequence with the first three terms missing.

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4. Find the ratio between two consecutive terms. _____
5. Find the first three terms.

### Think and Discuss

6. **Discuss** the similarities and differences between the two sequences.

_____

7. **Explain** your strategy for finding the 20th term in the first sequence.

_____

_____

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# Practice A

## *Geometric Sequences*

**Find the common ratio for each of the following geometric sequences.**

**1.** 5, 10, 20, 40, 80, …

_____

**2.** 3, 15, 75, 375, 1875, …

_____

**3.** 1, 6, 36, 216, 1296, …

_____

**4.** 0.5, 1.5, 4.5, 13.5, 40.5, …

_____

**5.** 810, 270, 90, 30, 10, …

_____

**6.** 72, 7.2, 0.72, 0.072, 0.0072, …

_____

**Each of the following is a geometric sequence. Find the next three terms of the sequence.**

**7.** 2, 10, 50, 250, 1250, …

_____

_____

**8.** 4, 24, 144, 864, 5184, …

_____

_____

**9.** 375, 75, 15, 3, 0.6, …

_____

_____

**10.** $\frac{1}{3125}, \frac{1}{625}, \frac{1}{125}, \frac{1}{25}, \frac{1}{5}, \ldots$

_____

_____

**11.** 1.8, 3.6, 7.2, 14.4, 28.8, …

_____

_____

**12.** 6804, 2268, 756, 252, 84, …

_____

_____

**Use the formula for finding the *n*th term of a geometric sequence $a_n = a_1 r^{n-1}$, where $a_1$ is the first term in the sequence and *r* is the common ratio.**

**13.** Find the 4th term of the geometric sequence with $a_1 = 2$ and common ratio 3.

_____

**14.** Find the 5th term of the geometric sequence with $a_1 = 25$ and common ratio $\frac{1}{5}$.

_____

**15.** Julie is doing an experiment. She is studying a cell that triples every hour. She started the experiment with 24 cells. How many cells are there at the end of 4 hours?

_____

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# Practice B

## *Geometric Sequences*

**Determine if each sequence could be geometric. If so give the common ratio.**

**1.** 4, 16, 64, 256, 1024, …

_____

**2.** 3, $\frac{3}{2}$, $\frac{3}{4}$, $\frac{3}{8}$, $\frac{3}{16}$, …

_____

**3.** 5, 10, 15, 20, 25, …

_____

**4.** 3, 18, 108, 648, 3888, …

_____

**5.** 1250, 125, 12.5, 1.25, 0.125, …

_____

**6.** 10, 15, 22.5, 33.75, 50.625, …

_____

**7.** 36, 12, 4, $\frac{4}{3}$, $\frac{4}{9}$, …

_____

**8.** 1440, 720, 240, 60, 12, …

_____

**9.** 9, 3, 1, 0.5, 0.25, …

_____

**Find the given term in each geometric sequence.**

**10.** 6th term; 25, 75, 225, 675, 2025, …

_____

**11.** 10th term; $a_1 = 320$, $r = 0.5$

_____

**12.** 9th term; 4.5, 9, 18, 36, 72, …

_____

**13.** 7th term; $a_1 = 0.02$, $r = 10$

_____

**14.** 12th term; $\frac{1}{1000}$, $\frac{1}{100}$, $\frac{1}{10}$, 1, 10, …

_____

**15.** 8th term; $\frac{3}{8}$, $\frac{3}{4}$, $\frac{3}{2}$, 3, 6, …

_____

**16.** In an experiment a population of flies triples every week. The experiment starts with 12 flies. How many flies will there be by the end of week 5?

_____

**17.** A small business earned $21 in its first month. It quadrupled this amount each month for the next six months. How much did the business earn in the 4th month?

_____

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# Practice C

## *Geometric Sequences*

**Find the next three terms of each geometric sequence.**

**1.** $a_1 = 3$, common ratio $= 7$

**2.** $a_1 = 800$, common ratio $= 0.4$

**3.** $a_1 = \frac{3}{8}$, common ratio $= 2$

**4.** $a_1 = 7.6$, common ratio $= 8$

**Find the first five terms of each geometric sequence.**

**5.** $a_1 = 250$, $r = 0.6$

**6.** $a_1 = 0.16$, $r = 5$

**7.** $a_1 = \frac{2}{81}$, $r = 6$

**8.** $a_1 = 0.004$, $r = 10$

**9.** $a_1 = 12$, $r = 9$

**10.** $a_1 = 320$, $r = \frac{1}{4}$

**11.** Find the 1st term of a geometric sequence with a 5th term 11.664 and a common ratio of 0.6.

**12.** Find the 1st and 7th terms of a geometric sequence with a 3rd term $\frac{4}{9}$ and a 4th term $\frac{8}{27}$.

**13.** Find the 1st term of a geometric sequence with a 10th term $-1024$ and $r = -2$.

**14.** Water is leaking from a water tower. On the first day two gallons were lost. The leak is getting progressively worse and the amount of water lost triples each day. How many gallons would be lost on the 8th day?

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# Reteach

## *Geometric Sequences*

In a **geometric sequence,** the ratio between terms is constant

This is a geometric sequence with a common ratio of $\frac{3}{1} = 3$.

$2, \underset{\frac{3}{1}}{6}, \underset{\frac{3}{1}}{18}, \underset{\frac{3}{1}}{54}, \underset{\frac{3}{1}}{162}, \ldots$

This is not a geometric sequence since there is no common ratio.

$2, \underset{\frac{2}{1}}{4}, \underset{\frac{1}{2}}{2}, \underset{\frac{2}{1}}{4}, \underset{\frac{1}{2}}{2}, \ldots$

**Complete to determine if each sequence is geometric.**

**1.** $\frac{1}{125}, \underset{\frac{5}{1}}{\frac{1}{25}}, \underset{\_\_\_\_}{\frac{1}{5}}, \underset{\_\_\_\_}{1}, \underset{\_\_\_\_}{5}, \ldots$

geometric? _____

**2.** $\frac{1}{8}, \underset{\frac{2}{1}}{\frac{1}{4}}, \underset{\_\_\_\_}{\frac{3}{8}}, \underset{\_\_\_\_}{\frac{1}{2}}, \underset{\_\_\_\_}{\frac{5}{8}}, \ldots$

geometric? _____

**3.** $4, \underset{\_\_\_\_}{-2}, \underset{\_\_\_\_}{1}, \underset{\_\_\_\_}{-\frac{1}{2}}, \underset{\_\_\_\_}{\frac{1}{4}} \ldots$

geometric? _____

You can use the common ratio to find any term in a geometric sequence.

$2, \underset{4}{8}, \underset{4}{32}, \underset{4}{128}, \ldots$ This geometric sequence has a common ratio of $\frac{4}{1}$ or 4.

| | |
|---|---|
| This is the 1st term of the sequence. | 2 |
| Multiply it by the common ratio for the 2nd term. | $2 \times 4 = 8$ |
| Multiply it by the square of the common ratio for the 3rd term. | $2 \times 4^2 = 32$ |
| Multiply it by the cube of the common ratio for the 4th term. | $2 \times 4^3 = 128$ |
| For the $n$th term, multiply 1st term by the common ratio raised to the $(n - 1)$ power. | $2 \times 4^{n-1}$ |

**Complete to find the indicated term of the geometric sequence 2, 8, 32, 128, …**

**4.** the 5th term

$2 \times 4^{\_} =$ _____

**5.** the 7th term

$2 \times 4^{\_} =$ _____

**6.** the 6th term

$2 \times 4^{\_} =$ _____

**7.** the 10th term

$2 \times \_\_\_ =$ _____

**8.** the 8th term

$2 \times \_\_\_ =$ _____

**9.** the 9th term

$2 \times \_\_\_ =$ _____

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# Reteach

## Geometric Sequences (continued)

To find the $n$th term, $a_n$, of a geometric sequence with common ratio $r$: $\boldsymbol{a_n = a_1 \cdot r^{n-1}}$

Find the 7th term of this geometric sequence: 3, 6, 12, 24, 48, …

$a_n = a_1 \cdot r^{n-1}$ Find $r$. $r = \frac{6}{3} = 2$

$a_7 = 3 \cdot 2^{7-1}$ Substitute $n = 7$, $a_1 = 3$, $r = 2$.

$a_7 = 3 \cdot 2^{7-1} = 3 \cdot 2^6 = 3 \cdot 64 = 192$ The 7th term is 192.

**Find the indicated term of each geometric sequence.**

**10.** 1, 10, 100, 1000, . . .
Find the 9th term.

$n = 9$, $a_1 = 1$, $r = \frac{10}{1} =$ _____

$a_9 = 1 \cdot$ _____$^{9-1} = 1 \cdot$ _____

$a_9 = 1 \cdot$ _____

$=$ _____

**11.** 1.1, 1.21, 1.331, 1.4641, . . .
Find the 7th term.

$n = 7$, $a_1 = 1.1$, $r = \frac{1.21}{1.1} =$ _____

$a_7 = 1.1 \times$ _____ $= 1.1 \times$ _____

$a_7 = 1.1 \times$ _____

$=$ _____

Which sequence has the greater 20th term? by how much?

| 1000, 1050, 1100, 1150, … | 2, 4, 8, 16, … |
|---|---|
| arithmetic sequence, $d = 50$ | geometric sequence, $r = 2$ |
| $a_{20} = 1000 + (20 - 1)50$ | $a_{20} = 2 \cdot 2^{20-1}$ |
| $a_{20} = 1000 + (19)50 = 1000 + 950$ | $a_{20} = 2 \cdot 2^{19} = 2 \cdot 524{,}288 = 1{,}048{,}576$ |
| $a_{20} = 1950$ | $a_{20} = 1{,}048{,}576$ |

The 20th term of this geometric sequence is greater by 1,046,626.

**Identify the nature of the sequences and find their 15th terms.**

**12.** 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, . . .

sequence is _____

_____

_____

$a_{15} =$ _____

**13.** $\frac{5}{2}$, 3, $\frac{7}{2}$

sequence is _____

_____

_____

_____ $a_{15} =$ _____

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# Challenge

## *What's That Sum?*

To find the sum of $n$ terms of a geometric sequence with common ratio $r$: $S_n = \frac{a_1 - a_1 r^n}{1 - r}$

Find the sum of the first 5 terms of the geometric sequence 5, 15, 45, …

$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ Find $r$. $r = \frac{15}{5} = 3$

$S_5 = \frac{5 - 5 \cdot 3^5}{1 - 3}$ Substitute $n = 5$, $a_1 = 5$, $r = 3$.

$S_5 = \frac{5 - 5 \cdot 243}{1 - 3} = \frac{5 - 1215}{1 - 3} = \frac{-1210}{-2} = 605$

So, the sum of the first 5 terms of the sequence is 605.

Check: 5 + 15 + 45 + 135 + 405 = 605

**Use the formula to find each sum. Check your work by using a calculator to actually add the terms.**

**1.** 32, 16, 8, …
Find the sum of the first 6 terms.

$S_6$ = _____

**Check:**

_____

_____

**2.** −3, 15, −75, …
Find the sum of the first 5 terms.

$S_5$ = _____

**Check:**

_____

_____

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# Problem Solving

## *Geometric Sequences*

**For Exercises 1–2, determine if the sequence could be geometric. If so, find the common ratio. Write the correct answer.**

**1.** Because of advances in technology, the value of a computer decreases rapidly. A computer that is worth $1000 when purchased, could be worth $800 in six months, $640 in a year, $512 in 18 months, and $409.60 in two years.

_____

**2.** A student works for a starting wage of $6.00 per hour. She is told that she can expect a $0.25 raise every six months.

_____

**3.** A piece of paper that is 0.01 inches thick is folded in half repeatedly. If the paper were folded 6 times, how thick would the result be?

_____

**4.** A vacuum pump removes one-half of the air in a container with each stroke. How much of the original air is left in the container after 8 strokes?

_____

**For the last several years, the cost of a college education has increased an average of 5% per year. Choose the letter for the best answer.**

**5.** If the in-state tuition at the University of Florida is $2256 per year, what will the tuition be in 10 years?

**A** $3174.24 **C** $3499.80

**B** $3333.14 **D** $3674.79

**6.** If it costs $3046 per year for tuition for a Virginia resident at the University of Virginia now, how much will tuition be in 8 years?

**F** $4183.26 **H** $4500.33

**G** $4286.03 **J** $4725.35

**7.** If it costs $25,839 per year in tuition to attend Northwestern University now, how much will tuition be in 5 years?

**A** $31,407.47 **C** $37,965.97

**B** $32,977.84 **D** $42,483.72

**8.** If you start attending Northwestern University in 5 years and attend for 4 years, how much will you spend in total for tuition?

**F** $142,138.61 **H** $131,911.36

**G** $135,370.12 **J** $169,934.88

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# Puzzles, Twisters and Teasers

## *Itching for Answers!*

**Find the given term in each sequence. Use the answers to solve the riddle.**

**1.** 12th term: 3, 6, 12, 24, 48 ... _____ **T**

**2.** 8th term: 1, 4, 16, 64, 256 ... _____ **S**

**3.** 9th term: −4, −2, 0, 2, 4 ... _____ **F**

**4.** 6th term: $\frac{1}{2}$, 1, 2, 4 ... _____ **E**

**5.** 7th term: 3, 6, 12, 24 ... _____ **L**

**6.** 5th term: 1, 1.5, 2.25, 3.375 ... _____ **C**

**7.** 10th term: $\frac{1}{3}, \frac{-1}{3}, \frac{1}{3}, \frac{-1}{3}, \frac{1}{3}$ ... _____ **K**

**8.** 6th term: 96, 48, 24, 12, 6 ... _____ **A**

**9.** 10th term: 5, −5, 5, −5, 5 ... _____ **R**

**10.** 6th term: 1, 2, 4, 8 ... _____ **M**


**Where should you never take a dog?**

To a

| ____ | ____ | ____ | ____ | ____ | ____ | ____ | ____ | ____ | ____ |
|---|---|---|---|---|---|---|---|---|---|
| 12 | 192 | 16 | 3 | 32 | 3 | −5 | $-\frac{1}{3}$ | 16 | 6144 |