For the following probability distribution find the Expected value, E(X) and the variance, Var (X)

| $x$ | 20 | 40 | 60 | 80 | 100 |
|---|---|---|---|---|---|
| $P(X=x)$ | 0.1 | 0.15 | a | 0.2 | 0.05 |

a = 1-(0.1+0.15+0.2+0.05)

| $x$ | 20 | 40 | 60 | 80 | 100 |
|---|---|---|---|---|---|
| $P(X=x)$ | 0.1 | 0.15 | 0.5 | 0.2 | 0.05 |
| $xP(X=x)$ | 2 | 6 | 30 | 16 | 5 |

$\Sigma xP(X=x) =$ 59 E(X) = 59

| | | | | | |
|---|---|---|---|---|---|
| $x^2P(X=x)$ | 40 | 240 | 1800 | 1280 | 500 |

$\Sigma x^2P(X=x) =$ 3860 Var(X) = $\Sigma x^2P(X=x) -$ E(X)$^2$

$3860-59^2 = 379$

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# Probability using the Binomial Distribution

Binomial Distribution models situations which:

- have 2 outcomes that are mutually exclusive

  ***bi***nomial

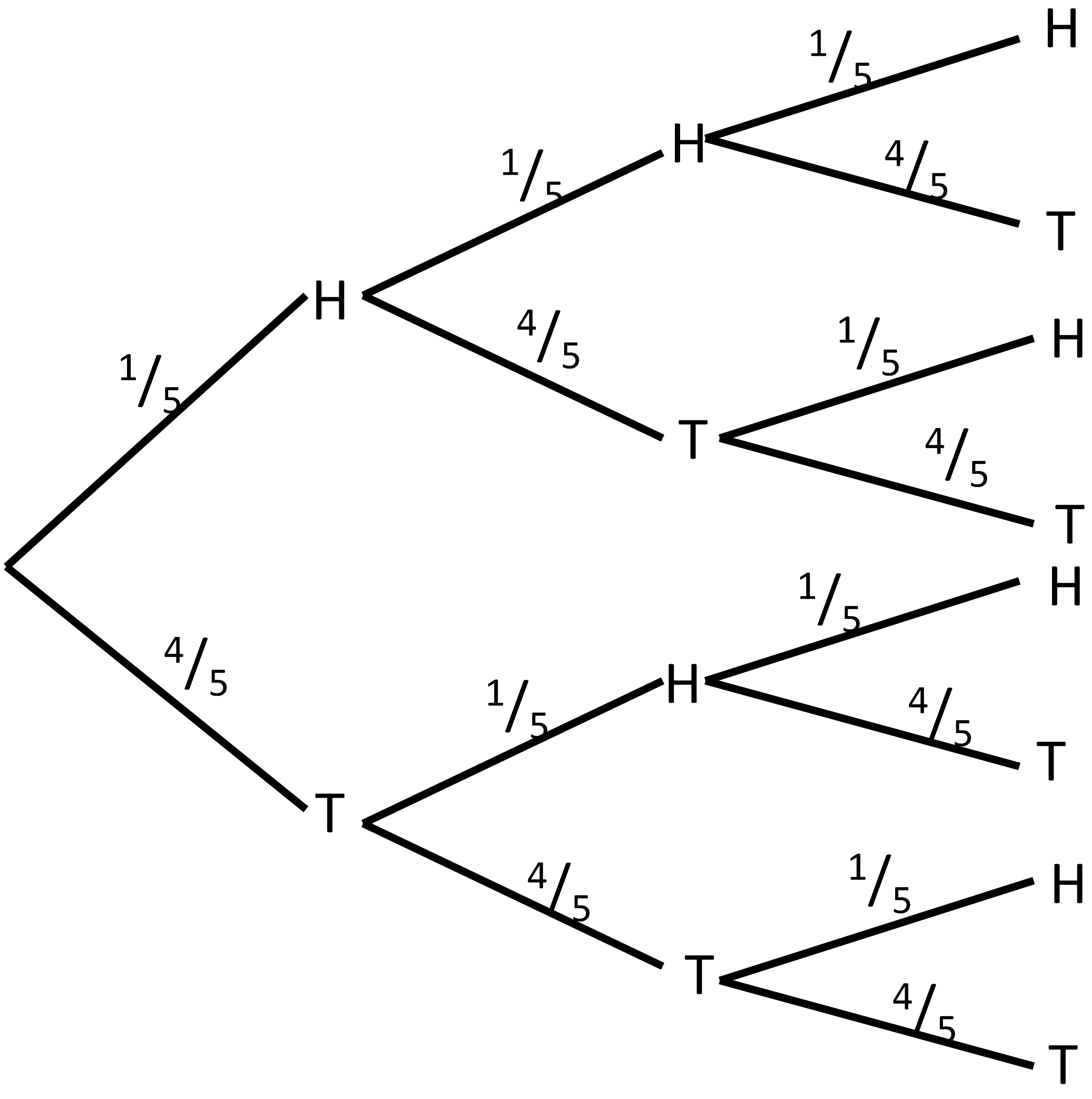
- are independent events
- have a finite number of events
- probabilities do not change with each successive event

It is used for determining the overall outcome of successive events where there is a finite number of trials.

Example:

Throwing a bent coin 3 times where $P(H) = {}^{1}/_{5}$

Find the probability of gaining 2 heads




| | |
|---|---|
| HHH | $\left(\frac{1}{5}\right)^3$ |
| HHT | $\left(\frac{1}{5}\right)^2 \times \left(\frac{4}{5}\right)$ |
| HTH | $\left(\frac{1}{5}\right)^2 \times \left(\frac{4}{5}\right)$ |
| HTT | $\left(\frac{1}{5}\right) \times \left(\frac{4}{5}\right)^2$ |
| THH | $\left(\frac{1}{5}\right)^2 \times \left(\frac{4}{5}\right)$ |
| THT | $\left(\frac{1}{5}\right) \times \left(\frac{4}{5}\right)^2$ |
| TTH | $\left(\frac{1}{5}\right) \times \left(\frac{4}{5}\right)^2$ |
| TTT | $\left(\frac{4}{5}\right)^3$ |

HHH $\left(\frac{1}{5}\right)^3$

HHT $\left(\frac{1}{5}\right)^2 \times \left(\frac{4}{5}\right)$

HTH $\left(\frac{1}{5}\right)^2 \times \left(\frac{4}{5}\right)$

HTT $\left(\frac{1}{5}\right) \times \left(\frac{4}{5}\right)^2$

THH $\left(\frac{1}{5}\right)^2 \times \left(\frac{4}{5}\right)$

THT $\left(\frac{1}{5}\right) \times \left(\frac{4}{5}\right)^2$

TTH $\left(\frac{1}{5}\right) \times \left(\frac{4}{5}\right)^2$

TTT $\left(\frac{4}{5}\right)^3$

P(3 Heads) = 1 x $\left(\frac{1}{5}\right)^3$

P(2 Heads) = 3 x $\left(\frac{1}{5}\right)^2 \times \left(\frac{4}{5}\right)$

P(2 Tails) =3 x $\left(\frac{1}{5}\right) \times \left(\frac{4}{5}\right)^2$

P(3 Tails) =1 x $\left(\frac{4}{5}\right)^3$

Notice the pattern 1 3 3 1

If the coin was thrown 4 times it would look like this:

P(4 Heads) $= 1 \times \left(\frac{1}{5}\right)^4$

P(3 Heads) $= 4 \times \left(\frac{1}{5}\right)^3 \times \left(\frac{4}{5}\right)$

P(2 Heads) $= 6 \times \left(\frac{1}{5}\right)^2 \times \left(\frac{4}{5}\right)^2$

P(1 Heads) $= 4 \times \left(\frac{1}{5}\right) \times \left(\frac{4}{5}\right)^3$

P(0 Heads) $= 1 \times \left(\frac{4}{5}\right)^4$

The number of ways of achieving a particular outcome can be calculated as:

$$^{n}C_{x}$$

Where **n** is the number of trials and $x$ is the number of required outcomes.

Example:

Using a biased coin where P(H) = 0.55 what is the probability of achieving 6 heads when the coin is thrown 8 times.

$$^{8}C_{6} \times 0.55^{6} \times 0.45^{2} \quad = 0.157$$

In general:

$$P(X=x) = {}^{n}C_{x} \times p^{x}q^{(n-x)}$$

Where

n = number of trials

p = probability of success

q = probability of failure

$x$ = number of required successes

A binomial distribution can be described as:

$$P(X=x) \sim B(n,p)$$

Example:

A card is to be drawn from a well shuffled pack and then replaced.

This is repeated 10 times.

What is the probability of drawing an Ace 6 times.

$$ {}^{10}C_6 \text{ x } \left(\frac{1}{13}\right)^6 \times \left(\frac{12}{13}\right)^4 = 3.159 \text{ x } 10^{-5} $$

What is the probability of drawing an Ace at least twice.

$$ 1 - P(X<2) \quad = \quad 1 - (P(X=0)+P(X=1)) $$

$$ 1 - \left({}^{10}C_0 \text{ x } \left(\frac{1}{13}\right)^0 \times \left(\frac{12}{13}\right)^{10} + {}^{10}C_1 \text{ x } \left(\frac{1}{13}\right)^1 \times \left(\frac{12}{13}\right)^9\right) $$

$$ = 0.177 $$

Example:

For the binomial distribution P(X=$x$) ~B(8,¼)

Find P(X=6)

$$^8C_6 \text{ x } \left(\frac{1}{4}\right)^6 \times \left(\frac{3}{4}\right)^2 = 0.00385$$

Now do these:

A random variable has a binomial probability distribution with $n = 10$ and $p = 0.3$

Find: $P(X=x) \sim B(10,0.3)$

a) $P(X=0)$ $\quad {}^{10}C_0 \times 0.7^{10} = 0.0282$

b) $P(X<9)$ $\quad$ 1- P(X=9)+P(X=10)

$1- {}^{10}C_{10} \times 0.3^{10} + {}^{10}C_9 \times 0.3^9 \times 0.7 = 0.9999$

A random variable has a binomial probability distribution with $n = 20$ and $p = 0.6$

Find: $P(X=x) \sim B(20,0.6)$

a) $P(X=10)$ $\quad {}^{20}C_{10} \times 0.6^{10} \times 0.4^{10} = 0.117$

b) $P(X>18)$ $\quad$ P(X=19)+P(X=20)

${}^{20}C_{19} \times 0.6^{19} \times 0.4 + {}^{20}C_{20} \times 0.6^{20} = 0.000524$

c) $P(12<X<15)$ $\quad$ P(X=13)+P(X=14)

${}^{20}C_{13} \times 0.6^{13} \times 0.4^7 + {}^{20}C_{14} \times 0.6^{14} \times 0.4^6 = 0.29$