

Binomial Expansion Examples

$$(a + b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$$

$$(1 + x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \dots + \binom{n}{n-1}x^{n-1} + \binom{n}{n}x^n$$

Example 1

- (a) The expression $(1 - 2x)^4$ can be written in the form $1 + px + qx^2 - 32x^3 + 16x^4$.
By using the binomial expansion, or otherwise, find the values of the integers p and q .

- (b) Find the coefficient of x in the expansion of $(2 + x)^9$.

(3)

- (c) Find the coefficient of x in the expansion of $(1 - 2x)^4(2 + x)^9$.

(2)

(3)

(Total 8 marks)

Example 2

1. (a) Obtain the expansion of $(1 - 2x)^3$

- (b) Hence write down the expansion of

$$\left(1 - \frac{2}{x}\right)^3$$

(2)

- (c) Find the constant term in the expansion of

$$(1 - 2x)^3 \left(1 - \frac{2}{x}\right)^3$$

(1)

(3)

(Total 6 marks)

Binomial Expansion Exercise

1. Find the coefficient of the term in x^4 in the binomial expansion of $(3 + 2x)^7$.

(Total 3 marks)

2. Find the coefficient of x^3 in the binomial expansion of $(2 + 3x)^9$. Give your answer as an integer.

(Total 3 marks)

3. (a) Write down all the terms in the binomial expansion of $(1 - x)^5$.

(2)

- (b) Find the coefficient of x^3 in the binomial expansion of $(3 - 2x)^5$. Give your answer as an integer.

(2)
(Total 4 marks)

4. Write down the expansion $(1 + x)^7$ in ascending powers of x up to and including the term in x^3 .

(3)

Hence determine the value of 1.00001^7 correct to 15 decimal places.

(2)
(Total 5 marks)

5. Find the binomial expansion of $\left(1 + \frac{1}{2}x\right)^{16}$ in ascending powers of x up to and including the term in x^3 .

(3)

Hence determine the coefficient of x^3 in the expansion of $(23 + 13x)\left(1 + \frac{1}{2}x\right)^{16}$.

(2)
(Total 5 marks)

6. (a) Write down the first four terms in ascending powers of x in the expansion of $(1 + x)^8$, simplifying your coefficients as much as possible.

(2)

- (b) Find the coefficient of x^3 in the expansion of $(3 - 2x)(1 + x)^8$.

(2)
(Total 4 marks)

Binomial Expansion Solutions

1.	$\left(\frac{7}{4}\right)^3 2^4$	M1 A1		
	15 120		A1	3
			[3]	
2.	$\left(\frac{9}{3}\right)^2 6^3$	M1 M1		
	145152		A1	3
			[3]	
3.	(a) $1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$	M1		
	(b) $-10 \times 3^2 \times 2^3 = -720$	M1		
	$3^5 \left(\frac{2}{3}\right)^2 \times 10$ or $\frac{5 \times 4 \times 3}{2 \times 3} \times 3^2 \times 2^3$ are acceptable	A1	2	
	<i>CAO</i>			
	<i>SC 1080x² 1/2</i>			[4]
4.	$(1+x)^7 \equiv 1 + 7x$	B1		
	$= 21x^2 + 35x^3$	B1 B1		3
	Substituting $x = 10^{-5}$	M1		
	$1.00001^7 = \frac{1.000070002100035}{(to\ 15\ decimal\ places)}$	A1		2
			[5]	
5.	$1 + 8x + 30x^2 +$ (attempt at least 3 terms) may be unsimplified $+ 70x^3$ (simplified)	M1 A1 A1 cao		3
	23 (coeff. of x^3) $+ 13$ (coeff. of x^2)	M1		
	$23 \times 70 + 13 \times 30 = \underline{2000}$	A1 cao		2
			[5]	
6.	(a) $1 + 8x + 28x^2 + 56x^3$	M1 A1		2
	(b) $28 \times -2 + 3 \times 56$ "their 28 and 56" $= 112$	M1 A1		2
			[4]	