N13 • Analysing sequences

Mathematical goals

To enable learners to:

- define a sequence using the general form of the nth term, e.g. $u_n = 2n^3$;
- define a sequence inductively, e.g. $u_{n+1} = 4u_n - 1$;
- recognise and define an arithmetic progression;
- recognise and define a geometric progression;

and to reflect on and discuss these processes.

These goals may be adapted for learners aiming at lower level qualifications. For example, you could leave out explicit references to arithmetic and geometric progressions.

Starting points

Learners should have some understanding of what a number sequence is.

Materials required

For each learner you will need:

- mini-whiteboard;
- Sheet 1 – *Arithmetic and geometric progressions*.

For each small group of learners you will need:

- Card set A – *Sequences* (two pages);
- Card set B – *Blanks*.

Time needed

At least 45 minutes.

Suggested approach

Learners aiming at lower levels can be asked to read out one of their descriptions. Then other members of the group have to write down the sequence (if possible) from the description and compare it with the original. As a result, descriptions may need to be rewritten.

Beginning the session

Explain that learners have to make as many number sequences as they can, using the numbers on the *Sequences* cards they will be given. Each sequence must contain at least three cards.

Working in groups (1)

Ask learners to work in pairs. Give out Card set A – *Sequences*.

Learners who find it easy to make sequences can be encouraged to create more complex sequences.

When plenty of sequences have been created, put some blank cards out on each table. For each sequence that learners have made (or a selection if they have a lot), ask them to copy the sequence onto a blank card and to add a description, in words, of the pattern of the sequence.

Whole group discussion (1)

Write some of the sequences on the board and discuss how to express them using mathematical notation, in particular u_n as the nth term.

For example,

"1, 5, 9 $\Rightarrow$ add 4 on each time" becomes:

$$u_1 = 1 \quad u_2 = u_1 + 4 \quad u_3 = u_2 + 4 \quad \ldots \quad u_{n+1} = u_n + 4$$

or

"1, 4, 9 $\Rightarrow$ square numbers" becomes:

$$u_1 = 1^2 \quad u_2 = 2^2 \quad u_3 = 3^2 \quad \ldots \quad u_n = n^2$$

Using mini-whiteboards, practise the definitions. Give learners an algebraic or an inductive definition and ask them to write down the first three terms, or the fifth term, etc. Giving an inductive definition without a starting term can highlight the need for a starting number when defining a sequence inductively.

Working in groups (2)

Give the pairs of learners some time to write inductive or nth term definitions for the sequences they have on their cards. At this stage, most of their definitions will probably be inductive so you can encourage them to write alternative algebraic definitions.

Whole group discussion (2)

Ask learners to write an algebraic or inductive definition of one of their sequences on the board and invite the rest of the group to give the sequence from that definition. This can then be compared with the original sequence to see if the definition is correct.

Define arithmetic and geometric progressions. Ask learners to go back to Card set A. Ask learners to:

- find an arithmetic progression that includes the number 20;
- find a geometric progression that includes the number 3;
- find the longest arithmetic progression that you can;
- find the longest geometric progression that you can.

Write some of the answers on the board and ask learners to find algebraic and inductive definitions for them.

Reviewing and extending learning

Ask each learner to complete Sheet 1 – *Arithmetic and geometric progressions*. When they have finished, they should swap with a partner who should comment on their progressions and explanations.

Learners can generalise their learning to obtain $u_n = a + (n - 1)d$ and $u_n = ar^{n-1}$ for arithmetic and geometric progressions respectively.

What learners might do next

Give at least one reason why each of the following sequences might be the odd one out, i.e. it has a property that none of the others has.

(a) 2, 8, 18, 32 . . .

(b) 2, 5, 8, 11 . . .

(c) 3, 6, 12, 24 . . .

(d) 2, 1, 0.5, 0.25 . . .

(e) 2, –6, 18, –54 . . .

For example: ‘(b) is an arithmetic sequence and the others are not.’

Give possible arithmetic progressions such that the sum of the first five terms is 260.

This session could be followed by work on sums of series.

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N13 Card set A – *Sequences* (page 1)

40	0.7	12	$\frac{1}{2}$
–2	18	12.5	27
0.9	6	54	25
1	15	4	3
–0.1	–3	20	8

N13 Card set A – *Sequences* (page 2)

$\frac{3}{2}$	$\frac{1}{4}$	36	$\frac{2}{3}$
–6	0.2	–20	–0.5
$\frac{1}{3}$	–10	$\frac{1}{6}$	$\frac{3}{4}$
60	$\frac{1}{8}$	37.5	17
9	0.1	51	0.3

N13 Card set B – *Blanks*

N13 Sheet 1 – *Arithmetic and geometric progressions*

Name: . . . .

Write down the first four terms of an arithmetic progression that has the number 100 as its second term.

Explain why it is an arithmetic progression.

Write down an inductive definition for this sequence.

Write down an algebraic definition for this sequence.

Write down the first four terms of a geometric progression that has the number 100 as its third term.

Explain why it is a geometric progression.

Write down an inductive definition for this sequence.

Write down an algebraic definition for this sequence.